ELIZADE UNIVERSITY, ILARA-MOKIN,

ONDO STATE

FACULTY OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

SECOND SEMESTER EXAMINATION, 2018/2019 ACADEMIC SESSION

COURSE TITLE: ENGINEERING MATHEMATICS IV

COURSE CODE: GNE 316

EXAMINATION DATE: 9th July, 2019

COURSE LECTURER(S): Prof. Momoh-Jimoh Salami, Dr. Akinwumi A. Amusan

TIME ALLOWED: 3 hours

HOD's SIGNATURE

INSTRUCTIONS:

1. ANSWER <u>ANY FIVE QUESTIONS</u>
2. ANY INCIDENT OF MISCONDUCT, CHEATING, POSSESSION OF UNAUTHORIZED MATERIALS DURING EXAM SHALL BE SEVERELY PUNISHED.
3. YOU ARE NOT ALLOWED TO BORROW CALCULATORS AND ANY OTHER WRITING MATERIALS DURING THE EXAMINATION.
4. ELECTRONIC DEVICES CAPABLE OF STORING AND RETRIEVING INFORMATION ARE PROHIBITED.
5. DO NOT TURN TO YOUR EXAMINATION QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Question #1 (12 Marks)

A periodic function, with period 2π, is described as

$$f(x) = \begin{cases} \pi^2, & -\pi < x < 0; \\ (x-\pi)^2, & 0 < x < \pi. \end{cases}$$

(a) Sketch a graph of $f(x)$ for $-3\pi \le x \le 3\pi$. (2)

(b) Compute its Fourier series expansion. (6)

(c) From the result obtained in (b), show that

(i) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (2)

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ (2)

Question #2 [12 Marks]

Consider a periodic function that is described by

$$f(x) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2}; \\ \pi - x, & \frac{\pi}{2} \le x \le \pi. \end{cases}$$

(a) Sketch the graph of $f(x)$ for $= -2\pi < x < 2\pi$ for the two cases where:

(i) $f(x)$ is an even function. (3)

(ii) $f(x)$ is an odd function. (3)

(b) Determine the Fourier series expansion for the case a(i). (4)

(c) From the result obtained in (b), show that (2)

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}.$$

Question #3 (12 Marks)

(a) A periodic function, with period 10, is defined as

$$f(x) = \begin{cases} 0, & -5 < x < 0; \\ 3, & 0 < x < 5. \end{cases}$$

(i) Sketch a graph of $f(x)$. (2)

(ii) Determine its Fourier series expansion. (4)

(b) Given vectors $\vec{A} = \mathrm{p}\mathrm{i} - 6\mathrm{j} - 3\mathrm{k}$; $\vec{B} = 4\mathrm{i} + 3\mathrm{j} - \mathrm{k}$; and $\vec{C} = \mathrm{i} - 3\mathrm{j} + 2\mathrm{k}$; determine

(i) the value of p for which vectors $\vec{A}$ and $\vec{B}$ are perpendicular (1)

(ii) the value of p for which vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are coplanar (2)

(c) Determine whether or not the following vector field is conservative. Determine the corresponding scalar field ϕ if conservative.

$$\vec{F} = i(2xy + z) + j(x^2 + 2yz) + k(x + y^2)$$ (3)

Question #4 (12 Marks)

Given that

$$\Gamma(y) = \int_0^\infty t^{y-1} e^{-t} dt = (y-1)!$$

Evaluate (6)

(i)

$$\int_0^\infty \sqrt{x}\, e^{-\sqrt{x}} dx$$

(ii)

$$\int_0^\infty x^6 e^{-2x} dx$$

(iii)

$$\int_0^1 (\ln x)^4 dx$$

(a) Suppose m and n are positive constants, show that (6)

$$\int_0^\infty x^m e^{-ax^n} dx = \frac{1}{na^{\left(\frac{m+1}{n}\right)}} \Gamma\left(\frac{m+1}{n}\right)$$

Question #5 [12 Marks]

Given that

$$B(m,n) = \int_0^1 y^{m-1}(1-y)^{n-1}dy = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$

(a) Evaluate (4)

(i)

$$\int_0^2 (4-x^2)^{3/2}\, dx$$

(ii)

$$\int_0^{\frac{\pi}{2}} cos^5\, \theta sin^2\theta d\theta$$

(b) Given that

$$\int_0^{\infty} \frac{x^{p-1}}{1+x} dx = \frac{\pi}{\sin p\pi},$$

(3)

Show that

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin p\pi}$$

(3)

Hence, evaluate

$$\int_0^2 x(8-x^3)^{\frac{1}{3}}\, dx$$

(2)

(c) Show that

$$\int_a^b e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}\{erfc(a) - erfc(b)\}$$

Question #6 (12Marks)

(a) Given that

$$x^2y'' + xy' + y = 0$$

Show that

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$ (2)

(b) Use the Leibnitz-Maclaurin method to determine the series solution for the equation (5)

$$(1+x^2)y'' + xy' - 9y = 0$$

(c) Use the Frobenius method to obtain the series solution to the differential equation (5)

$$3x^2y'' - xy' + y - xy = 0$$

Question #7 (12 Marks)

(a) Given that vector $\vec{A} = i(x^2y) + j(xy+yz) + k(xz^2)$; and $\vec{B} = i(yz) - j(3xz) + k(2xy)$, determine at point $(1,2,1)$

(i) $\vec{\nabla}.\vec{B}$ (1)

(ii) $\vec{\nabla} \times \vec{B}$ (2)

(iii) $\vec{\nabla}(\vec{\nabla}.\vec{A})$ (2)

(iv) $\vec{\nabla}.\left(\vec{\nabla} \times (\vec{A} \times \vec{B})\right)$ (1)

(b) Apply Green´s theorem to evaluate the integral

$$\oint_C [(x-y)dx - (y^2+xy)dy]$$

where C is the circle with unit radius, centered on the origin. (3)
(Note: $x = r\cos\theta$, $y = r\sin\theta$, $dxdy = rdrd\theta$)

(c) By the use of the divergence theorem, determine

$$\oint_S \vec{F}.\vec{dS}$$

where, $\vec{F} = i(x) + j(xy) + k(2)$, taken over the region bounded by plane $z = 0, z = 4$, $x = 0, y = 0$ and the surface $x^2 + y^2 = 9$ in the first octant. (3)
(Note: $x = \rho\cos\phi$ $y = \rho\sin\phi$, $z = z$, $dV = dxdydz = \rho d\rho d\phi dz$)